[2] $r=0$ @ $\theta = \frac{\pi}{6}, \frac{7\pi}{4}$

LOCAL MAX @ $\theta = \frac{\pi}{2}, \frac{17\pi}{12}$

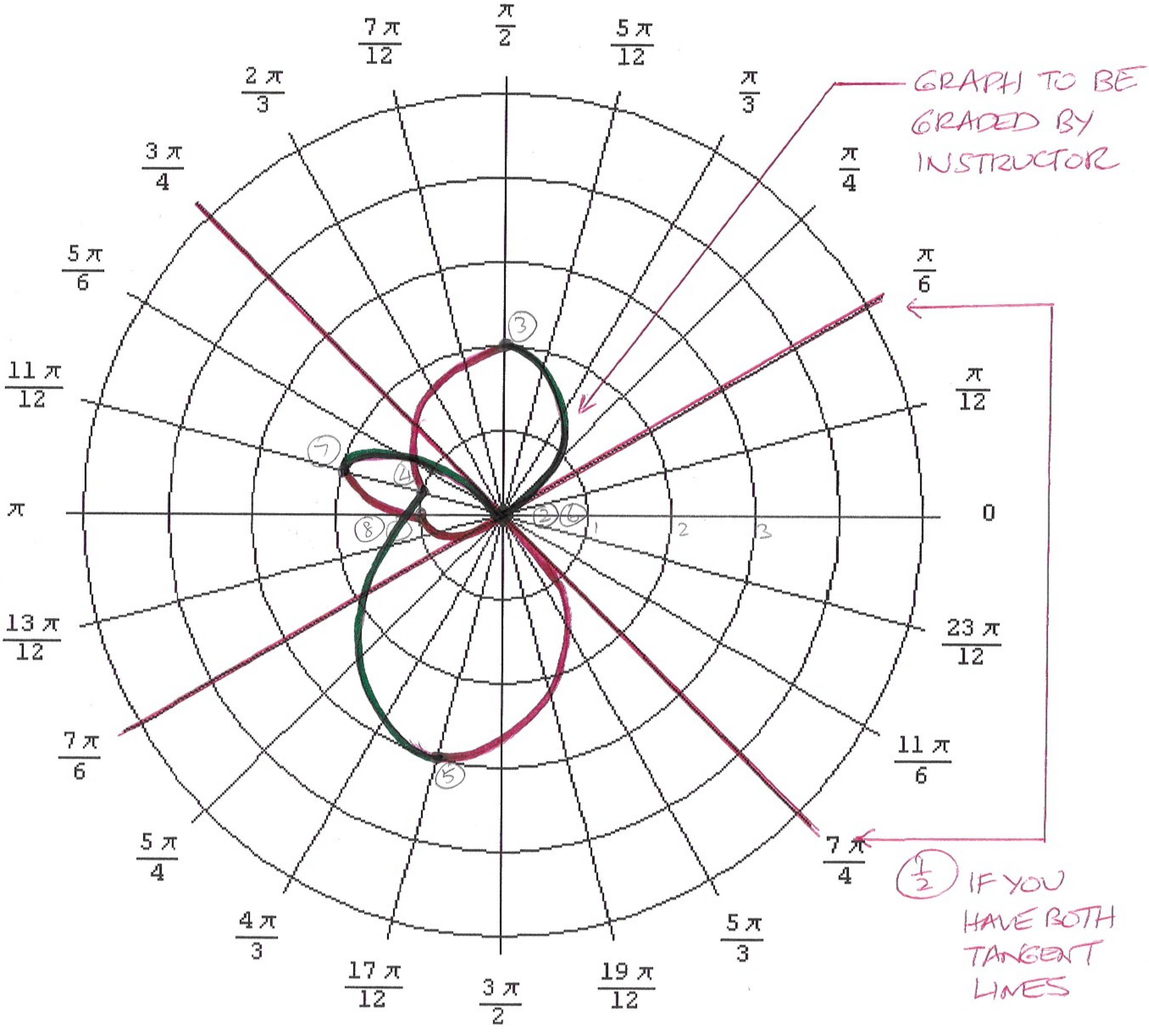
LOCAL MIN @ $\theta = \frac{11\pi}{12}, \frac{23\pi}{12}$

$r=-1$	$r=0$	$r=2$	$r=1$	$r=3$	$r=0$	$r=-2$	$r=-1$
0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{11\pi}{12}$	$\frac{17\pi}{12}$	$\frac{7\pi}{4}$	$\frac{23\pi}{12}$	2π
NEG POLAR AXIS	POLE	POS y-AXIS	Q_2	Q_3	POLE	Q_2	NEG POLAR AXIS

$1\frac{1}{2}$

$\lvert r \rvert$	DECR	INCR	DECR	INCR	DECR	INCR	DECR
SPIRAL	TOWARDS POLE	AWAY FROM POLE	TOWARDS	AWAY	TOWARDS	AWAY	TOWARDS

?



$\frac{\pi}{2}$
$\frac{5\pi}{12}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{\pi}{6}$
$\frac{\pi}{12}$
0
$\frac{23\pi}{12}$
$\frac{11\pi}{6}$
$\frac{7\pi}{4}$
$\frac{5\pi}{3}$
$\frac{19\pi}{12}$
$\frac{3\pi}{2}$
$\frac{17\pi}{12}$
$\frac{4\pi}{3}$
$\frac{5\pi}{4}$
$\frac{7\pi}{6}$
$\frac{13\pi}{12}$
π
$\frac{11\pi}{12}$
$\frac{5\pi}{6}$
$\frac{3\pi}{4}$
$\frac{2\pi}{3}$
$\frac{7\pi}{12}$
1
2
3
GRAPH TO BE GRADED BY INSTRUCTOR
$\frac{1}{2}$ IF YOU HAVE BOTH TANGENT LINES

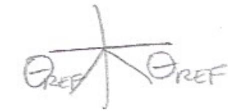
[3][a] $r=0$: $\boxed{\begin{aligned}-1+\sin\theta&=0\\ \sin\theta&=1\\ \theta&=\tfrac{\pi}{2}\end{aligned}}$ $\tfrac{1}{2}$

$(x,y)=(0,0)$

$\boxed{\begin{aligned}2+3\sin\theta&=0\\ \sin\theta&=-\tfrac{2}{3}\end{aligned}} \longrightarrow \theta\in Q_3, Q_4$

MUST HAVE BOTH

$\theta_{REF}=\sin^{-1}\frac{2}{3}$



$\boxed{\begin{aligned}\theta=\ &\pi+\sin^{-1}\tfrac{2}{3},\\ &2\pi-\sin^{-1}\tfrac{2}{3}\end{aligned}}$ 1

$\boxed{-1+\sin\theta=2+3\sin\theta}$

MUST HAVE BOTH

$-3=2\sin\theta$

$\boxed{\sin\theta=-\frac{3}{2}\ \text{IMPOSSIBLE}}$ $\tfrac{1}{2}$

$-r=-1+\sin(\pi+\theta)\longrightarrow -r=-1-\sin\theta\longrightarrow r=1+\sin\theta$

$\boxed{1+\sin\theta=2+3\sin\theta}$, $\theta\in[0,\pi]$

MUST HAVE BOTH

$-1=2\sin\theta$ $\tfrac{1}{2}$

$\boxed{\sin\theta=-\frac{1}{2}\ \text{IMPOSSIBLE}}$ IF $\theta\in[0,\pi]$

$-r=2+3\sin(\pi+\theta)\longrightarrow -r=2-3\sin\theta\longrightarrow r=-2+3\sin\theta$

$\boxed{-1+\sin\theta=-2+3\sin\theta}$, $\theta\in[0,\pi]$

MUST HAVE BOTH

$1=2\sin\theta$

$\boxed{\sin\theta=\frac{1}{2}\longrightarrow\theta=\frac{\pi}{6},\frac{5\pi}{6}}$ 1 ON CARDIOID / $\frac{7\pi}{6},\frac{11\pi}{6}$ ON LIMACON

CARDIOID

$\Theta = \frac{\pi}{6} \longrightarrow r = -1 + \sin\frac{\pi}{6} = -1 + \frac{1}{2} = -\frac{1}{2}$

$x = -\frac{1}{2}\cos\frac{\pi}{6} = -\frac{1}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{4}$

$y = -\frac{1}{2}\sin\frac{\pi}{6} = -\frac{1}{2}\cdot\frac{1}{2} = -\frac{1}{4}$

$(x,y) = \left(-\frac{\sqrt{3}}{4}, -\frac{1}{4}\right)$

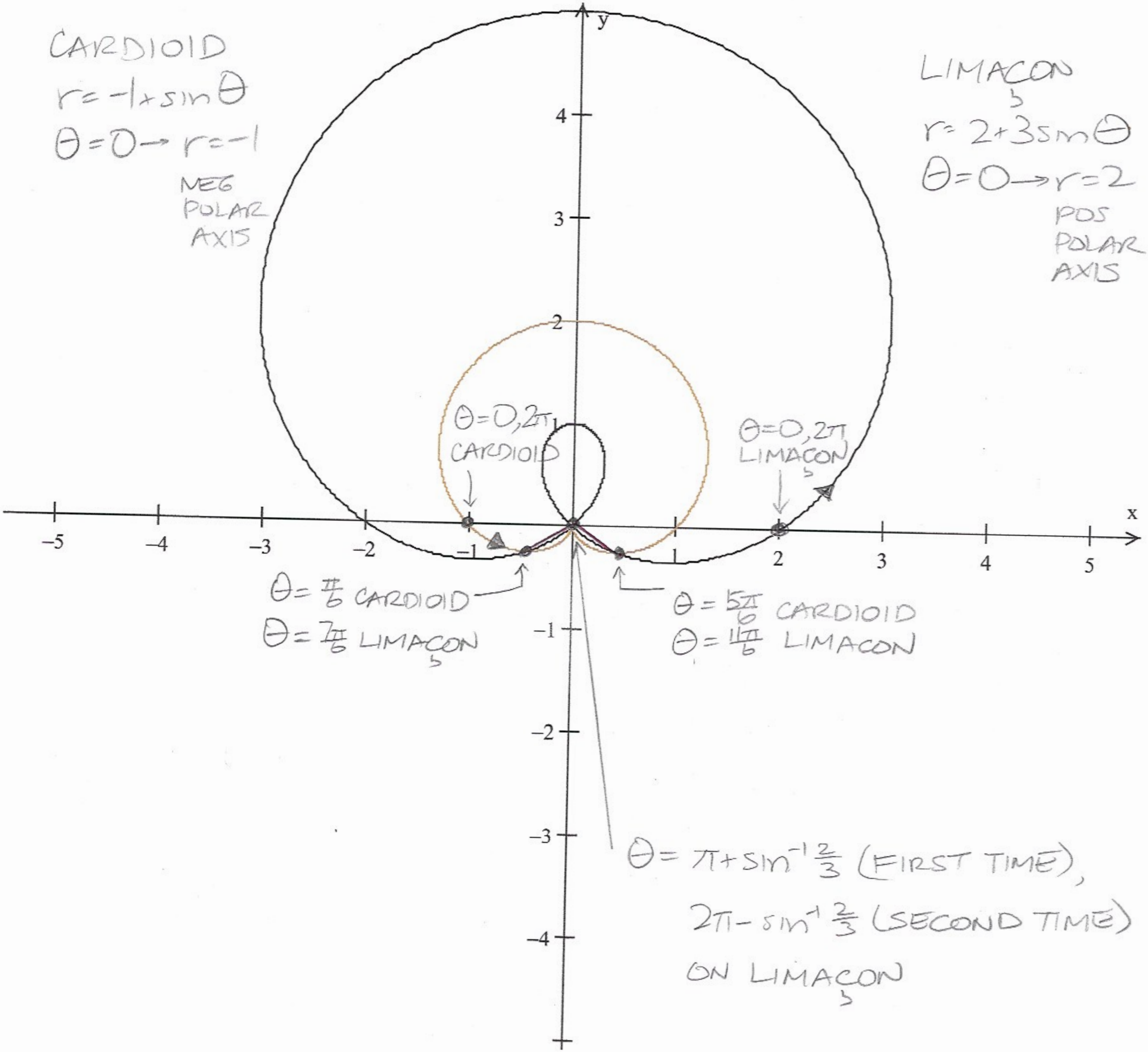
$\Theta = \frac{5\pi}{6} \longrightarrow r = -1 + \sin\frac{5\pi}{6} = -1 + \frac{1}{2} = -\frac{1}{2}$

$x = -\frac{1}{2}\cos\frac{5\pi}{6} = -\frac{1}{2}\cdot -\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$y = -\frac{1}{2}\sin\frac{5\pi}{6} = -\frac{1}{2}\cdot\frac{1}{2} = -\frac{1}{4}$

$(x,y) = \left(\frac{\sqrt{3}}{4}, -\frac{1}{4}\right)$

$\frac{1}{2}$



CARDIOID
r=-1+sinθ
θ=0→ r=-1
NEG POLAR AXIS
LIMAÇON
r=2+3sinθ
θ=0→r=2
POS POLAR AXIS
θ=0,2π CARDIOID
θ=0,2π LIMAÇON
θ=π/6 CARDIOID
θ=7π/6 LIMAÇON
θ=5π/6 CARDIOID
θ=11π/6 LIMAÇON
θ=π+sin⁻¹ 2/3 (FIRST TIME),
2π−sin⁻¹ 2/3 (SECOND TIME)
ON LIMAÇON
x
y
-5
-4
-3
-2
-1
1
2
3
4
5
-1
-2
-3
-4
1
2
3
4

[b] $x = (-1+\sin\theta)\cos\theta$

$y = (-1+\sin\theta)\sin\theta$

$$\frac{dy}{dx} = \boxed{\frac{\cos\theta\sin\theta + (-1+\sin\theta)\cos\theta}{\cos^2\theta - (-1+\sin\theta)\sin\theta}} \quad 1\tfrac{1}{2}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{5\pi}{6}} = \frac{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \left(-1+\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)^2 - \left(-1+\frac{1}{2}\right)\left(\frac{1}{2}\right)} = \frac{-\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}}{\frac{3}{4}+\frac{1}{4}} = \frac{0}{1} = \boxed{0} \quad \tfrac{1}{2}$$

$$\boxed{y - {-\tfrac{1}{4}} = 0\left(x - \tfrac{\sqrt{3}}{4}\right)} \quad \tfrac{1}{2}$$

$$y = -\frac{1}{4}$$

[c] $(2+3\sin\theta)^2+(3\cos\theta)^2 = 4+12\sin\theta+9\sin^2\theta+9\cos^2\theta$

$= 4+12\sin\theta+9 = 13+12\sin\theta$

$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$

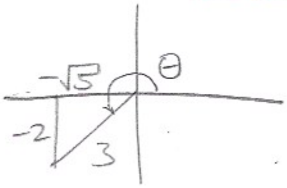
$$\boxed{\int_0^{\frac{7\pi}{6}}} \boxed{\sqrt{13+12\sin\theta}}\,d\theta + \boxed{\int_{\frac{11\pi}{6}}^{2\pi}} \boxed{\sqrt{13+12\sin\theta}}\,d\theta$$

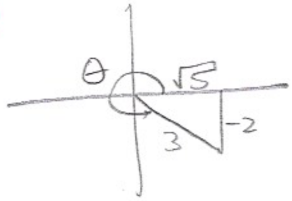
MUST HAVE BOTH

[d] $\boxed{\int_0^{\pi+\sin^{-1}\frac{2}{3}}} \ \frac{1}{2} \ \frac{1}{2} \boxed{\frac{1}{2}(2+3\sin\theta)^2} d\theta + \boxed{\int_{2\pi-\sin^{-1}\frac{2}{3}}^{2\pi}} \ \frac{1}{2} \ \boxed{\frac{1}{2}(2+3\sin\theta)^2} d\theta$

FOR $\theta = \pi + \sin^{-1}\frac{2}{3}$ FOR $\theta = 2\pi - \sin^{-1}\frac{2}{3}$

MUST HAVE BOTH





$$\int (2+3\sin\theta)^2 d\theta = \int (\boxed{4 + 12\sin\theta + 9\sin^2\theta}) d\theta \quad \frac{1}{2}$$

$$= 4\theta - 12\cos\theta + 9\cdot\frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$$

$$= 4\theta - 12\cos\theta + \frac{9}{2}\theta - \frac{9}{2}\sin\theta\cos\theta$$

$$= \boxed{\frac{17}{2}\theta - 12\cos\theta - \frac{9}{2}\sin\theta\cos\theta} \quad 1\frac{1}{2}$$

$$\left(\frac{17}{2}\theta - 12\cos\theta - \frac{9}{2}\sin\theta\cos\theta\right)\Big|_0^{\pi+\sin^{-1}\frac{2}{3}}$$

$$= \frac{17}{2}\left(\pi + \sin^{-1}\frac{2}{3}\right) - 12\left(-\frac{\sqrt{5}}{3}\right) - \frac{9}{2}\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) - (-12)$$

$$= \frac{17}{2}\pi + \frac{17}{2}\sin^{-1}\frac{2}{3} + 4\sqrt{5} - \sqrt{5} + 12$$

$$= \boxed{\frac{17}{2}\pi + \frac{17}{2}\sin^{-1}\frac{2}{3} + 3\sqrt{5} + 12} \quad \frac{1}{2}$$

$$\left(\frac{17}{2}\theta - 12\cos\theta - \frac{9}{2}\sin\theta\cos\theta\right)\Big|_{2\pi-\sin^{-1}\frac{2}{3}}^{2\pi}$$

$$= \frac{17}{2}\cdot 2\pi - 12 - \left(\frac{17}{2}\left(2\pi - \sin^{-1}\frac{2}{3}\right) - 12\left(\frac{\sqrt{5}}{3}\right) - \frac{9}{2}\left(-\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)\right)$$

$$= 17\pi - 12 - \left(17\pi - \frac{17}{2}\sin^{-1}\frac{2}{3} - 4\sqrt{5} + \sqrt{5}\right)$$

$$= \boxed{\frac{17}{2}\sin^{-1}\frac{2}{3} + 3\sqrt{5} - 12} \quad \frac{1}{2}$$

$$\text{AREA} = \frac{1}{2}\left(\frac{17}{2}\pi + \frac{17}{2}\sin^{-1}\frac{2}{3} + 3\sqrt{5} + 12 + \frac{17}{2}\sin^{-1}\frac{2}{3} + 3\sqrt{5} - 12\right)$$

$$= \frac{1}{2}\left(\frac{17}{2}\pi + 17\sin^{-1}\frac{2}{3} + 3\sqrt{5}\right)$$

$$= \boxed{\frac{17}{4}\pi + \frac{17}{2}\sin^{-1}\frac{2}{3} + 3\sqrt{5}} \quad \frac{1}{2}$$

[e]

$$\boxed{\int_0^{\frac{\pi}{6}} \frac{1}{2}(-1+\sin\theta)^2\,d\theta + \int_{\frac{5\pi}{6}}^{2\pi} \frac{1}{2}(-1+\sin\theta)^2\,d\theta}$$

1

EITHER ONE OK

1

$$\boxed{+\int_{\frac{7\pi}{6}}^{\pi+\sin^{-1}\frac{2}{3}} \frac{1}{2}(2+3\sin\theta)^2\,d\theta + \int_{2\pi-\sin^{-1}\frac{2}{3}}^{\frac{11\pi}{6}} \frac{1}{2}(2+3\sin\theta)^2\,d\theta}$$

OR

$$\boxed{\int_0^{2\pi} \frac{1}{2}(-1+\sin\theta)^2\,d\theta - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \frac{1}{2}(-1+\sin\theta)^2\,d\theta}$$

$$+\int_{\frac{7\pi}{6}}^{\pi+\sin^{-1}\frac{2}{3}} \frac{1}{2}(2+3\sin\theta)^2\,d\theta + \int_{2\pi-\sin^{-1}\frac{2}{3}}^{\frac{11\pi}{6}} \frac{1}{2}(2+3\sin\theta)^2\,d\theta$$